Functions

Problem. The number of cubic yards of dirt, D, needed to cover a garden with area a square feet is given by $D = g(a)$.

a. A garden with area 5000 ft^2 requires 50 cubic yards of dirt. Express this information in terms of the function g.

b. Explain the meaning of the statement $g(100) = 1$.

$D =$cubic yards of dirt.
$a =$Area of a garden in square feet.

$$50 = g(50000)$$

$g(100) = 1$. What does this mean?

$$D = g(a)$$
$$1 = g(100)$$

This means: For a garden with area 100 ft^2, we require 1 cubic yard of dirt.

Problem. Evaluate $f(3)$ and solve $f(x) = 1$.

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	74	28	1	53	56	3	36	45	14	47

Problem. Evaluate $f(2)$ for $f(x) = 8x^2 - 7x + 3$.

$$\begin{aligned} f(2) &= 8(2)^2 - 7(2) + 3 \\ &= 8 \cdot 4 - 14 + 3 \\ &= 32 - 14 + 3 = \textit{whatever} \end{aligned}$$

Problem. Find the domain of $f(x) = \frac{6}{x-8}$.

Possible answers: $x \neq 8$. $(-\infty, 8) \cup (8, \infty)$, "all reals except $x = 8$"

Linear Functions

Problem. At noon, a barista notices she has \$20 in her tip jar. If she makes an average of \$0.50 from each customer, how much will she have in her tip jar if she serves n more customers during her shift?

Look! This is a linear function! (the key here is the "rate of change" or slope: \$0.50 from each customer)
$T =$tips (in dollars)
$n =$customers

$$T = f(n) = \underbrace{b}_{initial} + \underbrace{m}_{slope} \cdot n$$
$$T = 20 + 0.50n$$

Problem. The cost in dollars to produce q tons of an item is given by the function $C = 100 + 20q$. What are the units of the 20?

$$\underbrace{C}_{\$} = \underbrace{100}_{\$} + \underbrace{20}_{\$/tons}\underbrace{q}_{tons}$$

Another way to do this: $C = \underbrace{100}_{initial} + \underbrace{20}_{slope} q$

The slope (aka rate of change) is always a... er... rate? (it needs to be a fraction)

Problem. Find the equation of a line that satisfies $f(-1) = 4$ and $f(5) = 1$.

Two points are enough to give us the equation of a line. $(\underbrace{-1}_{x_0}, \underbrace{4}_{y_0})$. $(5, 1)$

I like to use the point-slope equation of a line

$$y - y_0 = slope \cdot (x - x_0)$$
$$y - 4 = slope \cdot (x + 1)$$

To compute the slope we can do $\frac{1-4}{5-(-1)} = \frac{-3}{6} = -0.5$.
Long story short: $y - 4 = -0.5(x + 1)$.

If you decide to use the other point, you will get instead

$$y - 1 = -0.5(x - 5)$$

They look different, but they are the same!

Problem. A car rental company offers two plans for renting a car.

Plan A: 30 dollars per day and 18 cents per mile

Plan B: 50 dollars per day with free unlimited mileage

How many miles would you need to drive for plan B to save you money?

Plan A. $P = f(x) = b + m \cdot x$. $P = \$\$\$\$\$\$$, $x =$mileage.

$$P = f(x) = 30 + 0.18x$$

Plan B. $P = f(x) = 50$

$$\begin{cases} P &= 30 + 0.18x \\ P &= 50 \end{cases}$$

$$\begin{aligned} 30 + 0.18x &= 50 \\ 0.18x &= 50 - 30 \\ 0.18x &= 20 \\ x &= 20/0.18 \approx 111.11(miles) \end{aligned}$$

Quadratic functions, polynomials

Problem. I will give you the graph of a parabola, and ask that you find the equation. In example 6 from the book, the parabola is upside down.

First, find your vertex. Second, find your intersection with the y–axis. Third, write the parabola in the form

$$y = f(x) = a(x - p)^2 + q$$

The vertex is always (p, q). You have to find the value of a from the information of the y–intersect. For example, in problem 6, the vertex is at... $p = -2$, $q = 3$.

$$y = f(x) = a(x + 2)^2 + 3$$

At this point, evaluate this function at zero: $f(0) = a(0 + 2)^2 + 3 = 4a + 3$

You want that this expression is equal to the y–intersect! Solve in this case

$$\begin{aligned} 4a + 3 &= 2 \\ 4a &= -1 \\ a &= -1/4 = -0.25 \end{aligned}$$

Long story short: $y = -0.25(x + 2)^2 + 3$

Problem. Solve $(x + 3)(x + 1)^2(x - 4) > 0$

Exponential Functions and Logarithms

Problem. The amount, A (in mg), of a drug in the body is 25 when it first enters the system and **decreases** by 12% each hour. Find a formula for A as a function of t, in hours after the drug enters the system.

This seems to be an exponential function (I know because of the %). They are giving me the percentage rate of growth or decay (in this case decay).

$$\begin{aligned} r &= -0.12 \\ P_0 &= 25(mg) \end{aligned}$$

We have everything that we need. We have to write $P = P_0(1 + r)^t$.

$$P = 25(1 - 0.12)^t = 25(0.88)^t = 25 \cdot 88^t$$

Problem. Follow up question: When will the amount of drug in your system go below 5 mg?

For this question, all we have to do is solve the equation

$$\begin{aligned}
P(t) &= 5 \\
25(0.88)^t &= 5 \\
(0.88)^t &= \frac{5}{25} = 0.2 \\
0.88^t &= 0.2 \\
t &= \log_{0.88} 0.2 \\
t &\approx 12.5901233256
\end{aligned}$$

another way from the last expression:

$$\begin{aligned}
0.88^t &= 0.2 \\
\log 0.88^t &= \log 0.2 \\
t \cdot \log 0.88 &= \log 0.2 \\
t &= \frac{\log 0.2}{\log 0.88} \approx 12.5901233256
\end{aligned}$$

The amount of drug in your system will go below 5 mg in a little more than 12 hours and a half. (or if you prefer, just answer "in 12.59 hours")

Problem. Solve for t in the following equation:

$$200 = 30e^{0.15t}$$

$$\begin{aligned}
30e^{0.15t} &= 200 \\
e^{0.15t} &= \tfrac{20}{3} \\
0.15t &= \log_e \tfrac{20}{3} = \ln \tfrac{20}{3} \\
0.15t &= 1.897 \\
t &= 1.897/0.15 \\
t &= 12.647
\end{aligned}$$