Basic Integration:

$\int \sin x\,dx = -\cos x$ $\int \cot x\,dx = \ln|\sin x|$ $\int \sec^2 x\,dx = \tan x$ $\int \csc x \cot x\,dx = -\csc x$ $\int a^x\,dx = \frac{a^x}{\ln|a|}$ $\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x$

$\int \cos x\,dx = \sin x$ $\int \sec x\,dx = \ln|\sec x + \tan x|$ $\int \csc^2 x\,dx = -\cot x$ $\int \frac{1}{x}\,dx = \ln|x|$ Integration by Parts: $\int f(x)dx = \int a\,db = ab - \int b\,da$

$\int \tan x\,dx = -\ln|\cos x|$ $\int \csc x\,dx = -\ln|\csc x + \cot x|$ $\int \sec x \tan x\,dx = \sec x$ $\int e^x\,dx = e^x$ $\int \frac{1}{1+x^2}dx = \arctan x$ $\int \frac{1}{|x|\sqrt{x^2-1}}dx = \text{arcsec}\, x$

Trigonometric Integration:

- $\cos^2 x + \sin^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $\cot^2 x + 1 = \csc^2 x$
- $\sin x \times \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$
- $\cos x \times \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$
- $\sin x \times \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$

Trig. Subs.:

- $\sqrt{a^2 - x^2} \Rightarrow x = a\sin\theta$
- $\sqrt{x^2 - a^2} \Rightarrow x = a\sec\theta$
- $\sqrt{a^2 + x^2} \Rightarrow x = a\tan\theta$

$\sin(2x) = 2\sin(x)\cos(x)$ $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$

$\cos(2x) = \cos^2(x) - \sin^2(x)$ $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$

Integration of Rational Functions:

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{x^2 - a^2} = \frac{1}{(x-a)(x+a)} = \frac{A}{x-a} + \frac{B}{x+a} \qquad \int \frac{dx}{x^2 + a} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

Applications of Integrals:

$$A = \int_a^b \left(\begin{matrix}\text{upper}\\\text{function}\end{matrix}\right) - \left(\begin{matrix}\text{lower}\\\text{function}\end{matrix}\right)dx, \quad \text{or} \quad A = \int_c^d \left(\begin{matrix}\text{right}\\\text{function}\end{matrix}\right) - \left(\begin{matrix}\text{left}\\\text{function}\end{matrix}\right)dy, \quad \text{or} \quad A = \pi\left(\left(\begin{matrix}\text{outer}\\\text{radius}\end{matrix}\right)^2 - \left(\begin{matrix}\text{inner}\\\text{radius}\end{matrix}\right)^2\right)$$

$$Volume = \int_a^b \pi(f(x))^2\,dx$$

Sequences and Series:

$\sum_{n=1}^{\infty} \frac{1}{n} \rightarrow diverges$ $\quad$ $sum\ of\ series\ (geo\ \&\ p-series) = \frac{1st\ Term}{1-p}$ $\quad$ $sum\ for\ telescopic = \lim_{n\to\infty}(1st\ term - last\ term)$

Geo Series	P. Series	Ratio	Root	Limit Comp	Alternating	Integral	Divergence
$\sum_{n=0}^{\infty} ax^n$	$\sum_{n=0}^{\infty} \frac{1}{n^p}$	$\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert = L$	$\lim_{n\to\infty}\sqrt[n]{\lvert a_n\rvert} = L$	$\lim_{n\to\infty}\frac{a_n}{b_n} = L > 0$	$\sum_{n=1}^{\infty}(-1)^{n-1}a_n$	$\sum_{n=c}^{\infty} a_n\ (c \geq 0)$ $a_n = f(n)$	$\sum a_n$
CONV: $\lvert x\rvert < 1$, $\frac{a}{1-x}$ DIV: $\lvert x\rvert > 1$	CONV: $p > 1$ DIV: $p \leq 1$	ABS CONV: $0 < L < 1$ DIV: $1 < L < \infty$ INCON: $L = 1$		CONV: if $\sum b_n$ converge DIV: if $\sum b_n$ diverges	CONV: if b_n is decreasing $\lim_{n\to\infty} b_n = 0$	CONV: $\int_c^{\infty} f(n)$ converge DIV: $\int_c^{\infty} f(n)$ diverges	DIV: $\lim_{n\to\infty} a_n \neq 0$

integral test: decreasing ✓, positive ✓, and continuous ✓ $\qquad$ comparison test: choose series b so that series a< series b

Power Series:

$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$ $\quad$ $\cos x = \sum_{n=0}^{\infty}(-1)^n\frac{x^2}{(2n)!}$ $\quad$ $\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$ $\quad$ $\tan^{-1}x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)}$ $\quad$ $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

$f'(x) = \sum_{n=0}^{\infty} cn(x-a)^{n-1}$ $\quad$ $\int f(x)d(x) = \sum_{n=0}^{\infty}\frac{Cn}{n+1}(x-a)^{n+1}$ $\quad$ R= $\lim_{n\to\infty}\left\lvert\frac{an}{an+1}\right\rvert$ $\quad$ $Cn = \frac{f(a)(x-a)^n}{n!}$ $\quad$ $\lim_{n\to\infty}\sqrt{\frac{n}{n+1}} = 1$

$Taylor\ Series = (x-a)$ $\qquad$ $MacLaurin\ Series\ is\ centered\ at\ (0,0)$

Polar Coordinates:

$x = r\cos(\theta)$ $\quad$ $r^2 = x^2 + y^2$ $\quad$ $\tan(\theta) = \frac{y}{x}$ $\quad$ $A = \int_{\alpha}^{\beta}\frac{1}{2}f(x)^2\,d\theta$ $\quad$ $L = \int_{\alpha}^{\beta}\sqrt{f(\theta)^2 + f'(\theta)^2}\,d\theta$ $\quad$ $d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$

$y = r\sin(\theta)$ $\quad$ $r = \sqrt{x^2 + y^2}$ $\quad$ $\theta = \arctan(\frac{y}{x})$ $\quad$ $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$