Basic Integration:

-Integration by Parts: $\int sin^2x\,dx$

$= -sin(x)cos(x) + \int cos^2x\,dx \quad \leftarrow LOOP$

$= -sin(x)cos(x) + \int (1 - sin^2x)dx$

$= -sin(x)cos(x) + \int 1dx \;=\; 2\int sin^2x\,dx \;\rightarrow\; \int sin^2x\,dx = \frac{x - sin(x)cos(x)}{2}$

- Substitution: $\int tan\,x\,dx$

$= \int \frac{sin(x)}{cos(x)}dx$

$u = cos\,x \quad du = -sin\,x\,dx$

$-\int \frac{1}{u}du \;=\; -ln|cos\,x| + C$

- Substitution: $\int (cos^3x + 3cos^2x + 7\,cos\,x - 1)sin\,x\,dx$

$u = cos\,x, \quad du = -sin\,x\,dx$

$= -\int u^3 + 3u^2 + 7u - 1\,du$

$= -(\frac{cos^4x}{4} + cos^3x + \frac{7}{2}cos^2x - cos\,x)$

Trigonometric Integration:

$\int sin^3x\,cos^2x\,dx$

$= \int sin^2x\,cos^2x\,sin\,x\,dx$

$= \int (1 - cos^2x)cos^2x\,sin\,x\,dx$

$u = cos\,x$

$-du = +sin\,x\,dx$

$\int sin\,\underbrace{(2x)}_{A}\,cos\,\underbrace{(3x)}_{B}dx$

$= \int \frac{1}{2}[sin(2x - 3x) + sin(2x + 3x)]dx$

$= \frac{1}{2}\int [sin(-x) + sin(5x)]dx$

$u = -x \quad -du = dx$

$u = 5x \quad \frac{1}{5}du = dx$

$\int \frac{x^3}{\sqrt{4^2 - x^2}}dx$

$x = 4\sin\theta$

$dx = 4\cos\theta\,d\theta$

$\int \frac{(4\sin\theta)^3\,4\cos\theta}{\sqrt{16 - (4\sin\theta)^2}}d\theta \rightarrow 4^3\int \sin^3\theta\,d\theta$

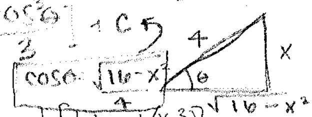
$= 4^3\int (1 - \cos^2\theta)\sin\theta\,d\theta$

$u = \cos\theta$

$-du = +\sin\theta\,d\theta$

$= -4^3\left[\cos\theta - \frac{\cos^3\theta}{3}\right] + C$

$x = 4\sin\theta \quad \sin\theta = \frac{x}{4} \quad \boxed{\cos\theta = \frac{\sqrt{16 - x^2}}{4}}$



Integration of Rational Functions:

$\int \frac{x^3 + 8x - 7}{x + 4}dx \quad x + 4\,\overline{)\,x^3 + 8x - 7} = x^2 - 4x + 24 - \frac{103}{x+4} \rightarrow \int x^2 - 4x + 24 - \frac{103}{x+4}$

Write as partial fraction: $\frac{5x+4}{(x+4)^3(x^2+5)^2} = \frac{A}{x+4} + \frac{B}{(x+4)^2} + \frac{C}{(x+4)^3} + \frac{Dx+E}{x^2+5} + \frac{Fx+G}{(x^2+5)^2}$

$\int \frac{dx}{(2x^2 - 12x + 26)}$ $\xrightarrow{\text{comp. sq.}}$ $2x^2 - 12x + 26 \Rightarrow 2((x-3)^2 + 4) \Rightarrow \int \frac{dx}{2[(x-3)^2 + 4]} = \frac{1}{2}\left[\frac{1}{2}tan^{-1}\left(\frac{x-3}{2}\right)\right] + C$

$\int \frac{3x-1}{x^2 + 10x + 28}dx$ complete square $x^2 + 10x + 28 \longrightarrow x^2 + 10x + 25 - 25 + 28 = (x+5)^2 + 3 \Rightarrow \int \frac{3x-1}{(x+5)^2 + 3}dx \quad u = x + 5 \quad x = u - 5 \quad du = dx$

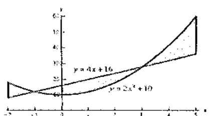
Applications of Integrals:

$y=2x^2+10$, $y=4x+16$, $x=-2$ and $x=5$

$A = \int_{-2}^{-1} 2x^2 + 10 - (4x+16)dx + \int_{-1}^{3} 4x + 16 - (2x^2 + 10)dx + \int_{3}^{5} 2x^2 + 10 - (4x+16)dx$

$= \int_{-2}^{-1} 2x^2 - 4x - 6dx + \int_{-1}^{3} -2x^2 + 4x + 6dx + \int_{3}^{5} 2x^2 - 4x - 6dx$

$= \left(\frac{2}{3}x^3 - 2x^2 - 6x\right)\Big|_{-2}^{-1} + \left(-\frac{2}{3}x^3 + 2x^2 + 6x\right)\Big|_{-1}^{3} + \left(\frac{2}{3}x^3 - 2x^2 - 6x\right)\Big|_{3}^{5}$

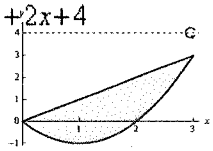


$y=x^2-2x$ and $y=x$ about the line $y=4$

inner radius $(4 - x)$

outer radius $= 4 - (x^2 - 2x) = -x^2 + 2x + 4$

$A(x) = \pi\left((-x^2 + 2x + 4)^2 - (4 - x)^2\right)$

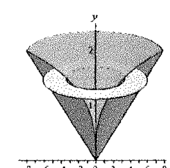


$y = \sqrt[3]{x}$ and $y=x/4$ about the y-axis

$y = \sqrt[3]{x} \quad \Rightarrow \quad x = y^3$

$y = \frac{x}{4} \quad \Rightarrow \quad x = 4y$

$A(y) = \pi\left((4y)^2 - (y^3)^2\right)$



Sequences and Series:

$\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)} \rightarrow \frac{1}{(n+2)(n+3)} = \frac{A}{(n+2)} + \frac{B}{(n+3)}$

$\rightarrow \sum_{n=1}^{\infty}\left(\frac{1}{(n+2)} - \frac{1}{(n+3)}\right)$

↑Telescopic series ↑

limit comparison test

$\sum_{n=0}^{\infty} \frac{3^n}{5^{n+1}+4} \rightarrow a_n = \frac{3^n}{5^{n+1}+4} \quad b_n = \frac{3^n}{5^{n+1}}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty}\left[\frac{3^n}{5^{n+1}+4} \cdot \frac{5^{n+1}}{3^n}\right]$

$= \lim_{n\to\infty} \frac{5^{n+1}}{5^{n+1}+4} = 1$

comparison test

$\sum_{n=0}^{\infty} \frac{1+sin(n)}{10^n} \quad a_n = \frac{1+sin(n)}{10^n}$

$1 - 1 < sin(n) + 1 < 1 + 1 \quad \rightarrow \quad 0 < sin(n) + 1 < 2$

$b_n > a_n \quad \rightarrow \quad \frac{3}{10^n} > \frac{1+sin(n)}{10^n}$

$\lim_{n\to\infty} \frac{3}{10^n} = 0$ *converges so...* $\sum_{n=0}^{\infty} \frac{1+sin(n)}{10^n}$ *also converges*

Power Series:

>Radius & interval of $\sum_{n=0}^{\infty} \frac{5^n(x+4)^n}{\sqrt{n}}$

1) lim| $\frac{5^{n+1}(x+4)^{n+1}}{\sqrt{n+1}} * \frac{\sqrt{n}}{5^n(x+4)^n}$ | → $\frac{5|x+4|\sqrt{n}}{\sqrt{n+1}}$

2) lim $\frac{5|x+4|\sqrt{n}}{\sqrt{n+1}}$ → 5|x+4| → 5|x+4|<1

3) |x+4|< $\frac{1}{5}$ R= $\frac{1}{5}$ 4)interval= [$\frac{-21}{5}$, $\frac{19}{5}$)

>Find MacLaurin Series of f(x)= $\frac{cos5x}{2}$

1)cos(x)= $\sum_{n=0}^{\infty}(-1)^n\frac{x^2}{(2n)!}$ → now plug in 5x for x

2) $\sum_{n=0}^{\infty}(-1)^n\frac{5x^2}{(2n)!}$ then / by 2 --> $\sum_{n=0}^{\infty}(-1)^n\frac{5x^2}{2(2n)!}$

>First 3 terms of Taylor Series of $e^x sinx$

>Eval $\int e^{-x^2}$ *as infinite series* 1) $e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} \rightarrow \int\sum\frac{(-1)^n x^{2n}}{n!} \rightarrow \sum\frac{(-1)^n x^{2n+1}}{(2n+1)n!}$

P.S for ln(1-x) and radius convergence

f'(x)= $\frac{-1}{1-x}$ = $-\sum x^n$ → $\int\frac{1}{1-x}$ = $\int -\sum x^n$ → ln(1-x)= $-\sum\frac{x^{n+1}}{n+1}$ ratio→ $\frac{|x|(n+1)}{n+2}$ =|x|<1 so R=1

: $e^x = 1 + x + \frac{x^2}{2!}$ sinx= $x - \frac{x^3}{3!} + \frac{x^5}{5!}$ = $x - x^2 + \frac{x^3}{3}$

>Find Power Series of f(x)= $\frac{1}{3-x}$: $\frac{1}{3} * \frac{1}{(1-\frac{x}{3})} \rightarrow \frac{1}{3}\sum_{n=0}^{\infty}(\frac{x}{3})^n$ or $\sum_{n=0}^{\infty}\frac{x^n}{3^{n+1}}$

Polar Coordinates:

$r = 5csc(\theta)$

$r = \frac{5}{sin(\theta)}$

$rsin(\theta) = 5$

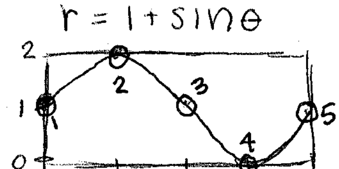
$r = 6\,sin(\theta) + 4\,cos(\theta)$

$r^2 = 6\,r\,sin(\theta) + 4\,r\,cos(\theta)$

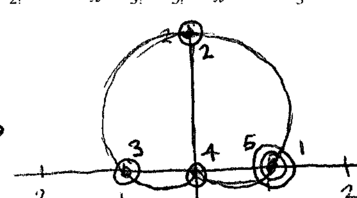
$x^2 + y^2 = 6x + 4y$

→ *complete the square* ←

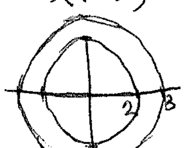
$r = 1 + \sin\theta$



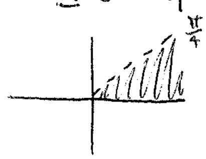
→



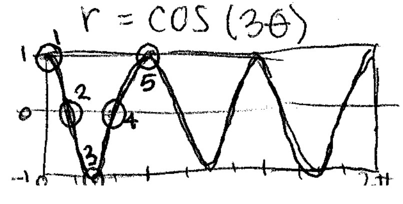
$(r-3)(r-2) = 0$



$0 \leq \theta < \frac{\pi}{4}$



$r = \cos(3\theta)$



→