Differential Equation Practice Test

Euler's Method (Paul Carter)

For the following problems, an initial value problem and its exact solution *y(x)* are given. Apply Euler's Method to approximate this solution on the interval [0,1] with step size *h = 0.25*.

1. $y' = y + 3\,,\ y(0) = 1\,;\ y(x) = 9e^{x^2} - 9$
2. $y' = 4xy^3\,,\ y(0) = 1\,;\ y(x) = \frac{1}{1-x^2}$

Slope Fields (Varsha Gopal)

Draw the slope field of the following equations:

3. $\frac{dy}{dx} = x + y$
4. $\frac{dy}{dx} = x^2 - y$

Draw the slope field for the following equation and use it to estimate the value of the solution:

5. $\frac{dy}{dx} = x + y,\ y(0) = 0,\ y(-4) = \ ?$
6. $\frac{dy}{dx} = y - x,\ y(4) = 0,\ y(-4) = \ ?$

First Order Differential Equations

-Separable (William Cromer)

7. $y' = 11x(y+3)^{\frac{3}{5}}$
8. $tan(x)y' = y - 1$
9. $y' = 3 + x + 3y + xy$
10. $\frac{dy}{dx} = \frac{(x^2-1)y^4}{x^3(2y^2-y)}$

-Exact (Benjamin Gray)

11. Find an implicit general solution to the following exact first order differential equation: $2xydx + (x^2 + 3y^2)dy = 0$

12. Find a general solution to the following first order differential equation: $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0$

13. Solve the following IVP: $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0$; $y(0) = \ -3$

14. Solve the following IVP and find the interval of validity: $\frac{2ty}{t^2+1} - 2t - (2 - ln(t^2 + 1))y' = 0$; $y(5) = 0$

15. Solve the following IVP: $3y^3e^{3xy} - 1 + (2ye^{3xy} + 3xy^2e^{3xy})y' = 0$; $y(0) = 1$

11: (https://www.math24.net/exact-differential-equations/#example1)
12-15: (http://tutorial.math.lamar.edu/Classes/DE/Exact.aspx)

-Linear (Maleesia Ragins)

Solve the the Linear first-order equations and solve for the IVP if given:

16. $2xy' + y = 10\sqrt{x}$
17. $xy' = 3y + x^4cos(x)$; $y(2\pi) = 0$

-Substitutions

•Homogeneous Differential Equations (C. Adam Gunter)

Find the general solutions of the following differential equations:

18. $2xyy' = x^2 + 2y^2$
19. $xy' = y + 2\sqrt{xy}$
20. $x^2y' = xy + x^2e^{\frac{y}{x}}$
21. $xyy' = y^2 + x\sqrt{4x^2 + y^2}$
22. $yy' + x = \sqrt{x^2 + y^2}$

(Hint: think v-sub; the y's need to go, the v's need to stay)

•Bernoulli Equations (Alan Rowland)

Find the general solutions of the following differential equations:

23. $xy' + y = y^2 ln(x)$
24. $y' + \frac{y}{3} = e^x y^4$
25. $xy' + y = y^2x^2 ln(x)$
26. $y' + \frac{2}{x}y = -x^2 cos(x) y^2$

(Remember: substitute with $v = y^{1-n}$, where n is the power of the non-linear y term)

23:(http://blancosilva.github.io/course/2018/08/13/ma242-fall-2018.html) - *Number 1 on the Bernoulli HW*

*24-26:(*http://www.cse.salford.ac.uk/physics/gsmcdonald/H-Tutorials/Bernoulli-differential-equations.pdf) - *Exercises 4, 8, and 6 respectively.*

•Jazz Equations (Chris Carter)

Find a general solution for the following problems:

27. $y' = x^2 + 3y$
28. $y' = (2y - 3x + 7)^3$
29. $y' = (5y + 4x^2 - 15)^{2/3}$
30. $y' = \frac{y+x+7}{y-x-7}$
31. $y' = \frac{(3y-2x^2+4)^2}{3y-2x^2}$

Second Order Differential Equations (Alex Stevens)

32. Find a general solution for: $y'' = -x(y')^2$

33. Find a general solution for: $y'' + y' = e^x$

34. Use the Wronskian to show that $y_1 = \sqrt{x}$ and $y_2 = x^{-1}$ are particular solutions of the differential equation $2x^2y'' + 3xy' - y = 0$

35. Solve the differential equation $y'' + 3y' + 2y = 4e^x$ using variation of parameters.

36. Solve the differential equation $y'' + 3y' + 2y = 4e^{-2x}$ that satisfies $y(0) = 0$ and $y'(0) = 2$ using the method of undetermined coefficients. (Solve for all relevant coefficients.)

37. Find a general solution for $y'' + 4y + 3 = 2x$ using the method of undetermined coefficients.

Second Order Differential Equations (Non Homogenous) (Julie Roark)

Find a general solution for the following problems:

38. $y'' + 4y' + 4y = t^2e$, $\quad t > 0$
39. $x'' + 9y = 10cos(2t)$
40. $y'' - 2y' - 3y = 5cos(2t)$
41. $y'' - 6y' - 7y = 8e^{-2t} - 7t - 6$
42. $y'' + 4y = 8cos(2t)$

http://www.math.psu.edu/tseng/class/Math251/Notes-2nd%20order%20ODE%20pt2.pdf

Geometric Applications 1 (John Michael Wager)

43. At each point of a curve, the intercept of the tangent line on the y-axis is equal to $4xy^2$. Find the curve s.

44. At each point of a curve, the subnormal is twice the square of the abscissa (x-coordinate). Find the curve if it also passes through the point (1, e).

45. Find orthogonal trajectories to the family of curves given by the equation:

    $y = ke^{-x} + 2x - 3$

Geometric Applications 2 (Thomas Stoback)

46. At Each point of a curve, the subnormal is twice the ordinate. Find the curve if it passes
    through the point ($3,\Pi$).

47. Find the equation of the curve where the slope of the tangent at any point is half the distance of the point to the x-axis.

48. Find the orthogonal trajectory of the curve $y = \sqrt[k]{x}$

49. Find the orthogonal trajectory of the curve $y = x - kx^n$ where n is a positive integer.

Population Models 1 (John Katorkas) Pg 82/83

50. The time rate of change of a rabbit population P is proportional to the square root of P. At time t = 0 months, the population numbers 100 rabbits and is increasing at the rate of 20 rabbits per month. How many rabbits will there be one year later?

51. The time rate of change of an alligator population P in a swamp is proportional to the square root of P. The swamp contained a dozen alligators in 1988. Two dozen in 1998. When will there be four dozen alligators in the swamp? What happens thereafter?

52. As the Salt KNO_3 dissolves in methanol, the number x(t) of grams of salt in the solution after t seconds satisfies the differential equation: $\frac{dy}{dx} = 0.8x - 0.004x^2$. What is the maximum amount of salt that will ever dissolve in the methanol. If x = 50 when t = 0, how long will it take for an additional 50g of salt to dissolve?

53. Suppose that a community contains 15000 people who are susceptible to Michaud's Syndrome, a contagious disease. At time t = 0, the number N(t) of people who have developed Michaud's is 5000 and is increasing at a rate of 500 a day. Assume N'(t) is proportional to the product of the numbers of those who have caught the disease and those who have not. How long will it take for another 5000 people to contract Michaud's?

54. During the period from 1790 to 1930 the US population P(t) (t in years) grew from 3.9 million to 123.2 million. Throughout this period, P(t) remained close to the solution of the initial value problem :

$$\frac{dy}{dx} = 0.03135P - 0.0001489P^2; \qquad P(0) = 3.9$$

What 1930 population does this logistical equation predict? What limiting population does it predict? Has this logistical equation continued since 1930 to accurately model the US population?

Needed Equations:
dP/dt = K*sqrt(P)
dP/dt = kP^2
dP/dt = KP(M-P)

Population Models 2 (Nicolas Davis)

55. Consider a logistic population p(t) or fish on a lake, measured in hundreds after t years, with k=3 and M=6. Suppose that 450 fish are harvested annually (at a constant rate throughout the year). If the lake is initially stocked with 375 fish, when will its population reach 90% of the carrying capacity?

56. Consider an animal population P(t) with constant death rate □=0.01 (deaths per animal per month) and with birth rate β proportional to P. Suppose that P(0)=200 and P'(0)=2. (a) When is P=1000? (b) When does doomsday occur? Pg(83) Q(27)

57. Consider a population P(t) satisfying the extinction-explosion equation $\frac{dP}{dt} = aP^2 - bP$, where $B = aP^2$ is the time rate at which births occur and $D = bP$ is the rate at which deaths occur. If the initial population is P(0)=Po and Bo births per month and Do deaths per month are occuring at time t=0, show that the threshold population is $M = \frac{D_0 P_0}{B_0}$. Pg(83) Q(18)

58. Suppose that the population P(t) of a country satisfies the differential equation dP/dt=kP(200-P) with k constant. Its population in 1960 was 100 million and was then growing at the rate of 1 million per year. Predict this country's population for the year 2020. Pg(83) Q(21)

2nd Law of Motion - Acceleration/Velocity (Chris Godwin)

59. A cannonball is fired straight upward from the ground with an initial velocity of 50 m/s. Find (a) The maximum height of the ball; (b) The total time the ball is in the air.

60. An 18 wheeler leaves a warehouse facility weighing 65,000 lbs. The motor provides an acceleration of $15\ m/s^2$, while the air resistance on the truck and trailer is $0.5\ m/s^2$ of deceleration for each meter per second of the 18 wheelers velocity. Find (a) The limiting velocity of the 18 wheeler; (b) How long it takes to reach the limiting velocity; (c) How far the 18 wheeler travels in that time span it takes to reach the limiting velocity.

2nd Law of Motion - Mechanical Vibrations (Chris Godwin)

Free - Undamped Motion

61. A mass of 10 kg is attached to the end of a spring stretched 50 cm by a force of 100 N. The mass is set in motion with an initial position x_0 = 0 cm and an initial velocity v_0 = 5 m/s. Find the amplitude, period and frequency of the resulting motion.

Free - Damped Motion

62. Supposed that the mass in a mass-spring-dashpot system with m = 20, c = 10 and k = 5 is set in motion with x_0 = 0 and v_0 = 10. Find the position function x(t) and determine whether the motion is overdamped, critically damped or underdamped.

Equations:

- Overdamped $c^2 > 4km$
- Critically damped $c^2 = 4km$
- Underdamped $c^2 < 4km$

Undamped - Forced Oscillations

63. Given that m = 2, k = 18, F_0 = 100 and ω = 10; this information gives the differential equation: $2x'' + 18x = 100cos10t$. Find the position function x(t), if x_0 = 0 and v_0 = 5.

Damped - Forced Oscillations

64. Given that m = 4, k = 16, c = 8, F_0 = 50 and ω = 5; this information gives the differential equation: $4x'' + 8x' + 16x = 50cos5t$. Find the position function x(t), if x_0 = 0 and v_0 = 0.

Manifest Of Human Things

Nicolas Davis - Population Model (803-847-4235).

John Katorkas - Population Model (803-468-7908)

Benjamin Gray - Exact (803-795-5001)

Julie Roark- Second Order Non-Homogeneous (864-425-6747)

Clinton Adam Gunter - First Order homogeneous

William Cromer - Separable first order

Thomas Stoback - Geometric Applications

Alan Rowland - Bernoulli

Paul Carter - Euler's Method

Maleesia Ragins-Linear first order

Varsha Gopal - Slope Fields

Alex Stevens - Second Order Differential Equations

Chris Godwin - 2nd Law of Motion (acceleration/velocity and mechanical vibrations)

Chris Carter - Jazz Equations

Equation Sheet

First Order Differential Equations

-Separable (William Cromer)

-Exact (Benjamin Gray)

$$\Psi_x + \Psi_y \frac{dy}{dx} = 0$$

$$\frac{d}{dx}\left(\Psi\left(x, y\left(x\right)\right)\right) = 0$$

Provided Ψ(x,y) is continuous and its first order derivatives are also continuous we know that:

$$\Psi_{x\,y} = \Psi_{y\,x}$$

$$\Psi = \int \Psi_x \, dx \quad \text{AND} \quad \Psi = \int \Psi_y \, dy$$

$$\Psi\left(x, y\right) = \int \Psi_x \; dx = a + h\left(y\right) \quad \text{AND} \quad \Psi\left(x, y\right) = \int \Psi_y \; dy = b + h\left(x\right)$$

$$\Psi\left(x, y\right) = a + h\left(y\right) = b + h\left(x\right)$$ where h is the difference in terms

$$\Psi(x, y) = c$$ -----Not quite, need to edit some more - Ben