Mechanical Vibrations: Wesley, Rich, John, Andrew

Free Undamped

1 A mass of **3 kg** is attached to the end of a spring that is stretched **20 cm** by a force of **15 N**. It is set in motion with initial position $\boldsymbol{x}(\mathbf{0}) = 0$ and initial velocity $\boldsymbol{v}(\mathbf{0}) = 10$ m/s. Find the amplitude, period, and frequency of the resulting motion. (*Rich Marini, pg. 181 #3*)

2 A body with mass **250g** is attached to the end of a spring that is stretched **20 cm** by a force of **9 N**. At time **t**=0 the body is pulled **1 m** to the right stretching the spring and set in motion with initial velocity of **5 m/s** to the left. Find **x(t)** in the form $\boldsymbol{C} * \boldsymbol{cos}(\boldsymbol{\omega}(\mathbf{0}) - \boldsymbol{r})$. Find the Amplitude and period of motion of the body. *(John Rickard pg.181 #4)*

3 Suppose that the mass in a mass spring dashpot system with **m**=24**, c**=10**,** and **k=**226 is set in motion with $\boldsymbol{x}(\mathbf{0}) = 20$ *and* $\boldsymbol{x}'(\mathbf{0}) = 41$.

A) Find the position function ***x(t)***

B) Find the pseudo-period of the oscillations and the equations of the "envelope curves." (*Andrew*)

Free Damped

4 Suppose that the mass of in a mass-spring-dashpot system with **m**=10, **c**=9, and k=2 is set in motion with $\boldsymbol{x}(\mathbf{0}) = 0$ and $\boldsymbol{dx/dt}(\mathbf{0}) = 5$.

A) Find the position function ***x(t)***

B) Find how far the mass moves to the right before starting backward toward the origin. *(John Rickard pg.182 #13)*

5 This problem deals with a high simplified model of a car of weight **3200 lb** (*mass m= 100 slugs in fps units*). Assume that the suspension system acts like a single spring and its shock absorbers like a single dashpot, so that its vertical vibrations satisfy the equation, $\boldsymbol{m} * \boldsymbol{a} + \boldsymbol{c} * \boldsymbol{v} + \boldsymbol{k} * \boldsymbol{x} = 0$ with appropriate values of the coefficients.

A) Find the stiffness coefficient k of the spring if the car undergoes free vibrations at 80 cycles per minute when its shock absorbers are disconnected. With the shock absorbers the car is set into vibration by driving over a bump, and the resulting damped vibration have a frequency of 78 cycle/min.

B) After how long will the time varying amplitude by 1% of its initial value? (*John Rickard pg. 182 #23*)

Forced Undamped

6 During a physics experiment, a pair of lab partners attached a **30cm** long spring to a **5kg** mass, and measured that it had stretched to a length of **1.8m**. Afterward, the spring was attached to a rotor that delivered a force to the spring with a force $\boldsymbol{6cos8t}$**.** Write and find an actual solution for the equation of spring in this experiment.

7 Consider the IVP (*3.6 #4*)
$\boldsymbol{x''} + \mathbf{25}\boldsymbol{x} = \mathbf{90}\boldsymbol{cos4t}$ where $\boldsymbol{x}(\mathbf{0}) = 0, \boldsymbol{x}'(\mathbf{0}) = 90$
Find the position function ***x(t)*** and the value of the constants. (*Andrew*)

Forced Damped

Find the actual solution of the given equations in the form of $x(t) = x_{sp}(t) + x_{tf}(t)$
$x'' + 18x' + 24x = 15sin(4t)$ where $x(0) = 15 = x'(0)$
$x'' + 7x' + 30x = 6cos(8t)$ where $x(0) = 5. x'(0) = 0$

La Place: Lindsay, Ryan, Trent, Matt, Ben

- Matt doing Laplace Transform of Derivatives
- Ben doing complex inverse problems
- Trent doing inverse, initial value and regular transformation problems

Laplace

Complex Inverse Examples (Ben)

$$F(s) = (2s - 3)/(s - 1)(s\text{^}2 + 4)\ (s > 1)$$

$$F(s) = (5s - 4)/(s\text{^}3 - s\text{^}2 - 2s)\ (s > 2)$$

$$F(s) = (2s\text{^}2 + s + 13)/(s - 1)((s + 1)\text{^}2 + 4)$$

Inverse Examples

$$F(s) = 3/s\text{^}4\ , (s > 0)$$

$$F(s) = 1/s(s + 1)(s + 2)\ , (s > 0)$$

Transformation

$$x\text{^}4e\text{^}(\pi x)$$
$$e\text{^}(-2x)sin(3\pi x)$$

Laplace Transform of Derivatives
Pg. 457 for number 1.

$$L\{t * cos(kt)\} = \frac{s^2 - k^2}{(s^2+k^2)^2} \quad for\ s > 0$$

$$L\{5 * sin(3x)\} = \frac{45}{s^2 + 9} \quad for\ s > 0$$

Transformation of initial value problems
Pg456

$$x\hat{}(ii) + 4x = 0 \ ; \ x(0) = 5, x\hat{}i(0) = 0$$

$$x\hat{}(ii) + x = cos(3t) \ ; \ x(0) = 1 \ , x\hat{}i(0) = 0$$

Acceleration/Velocity: Jacob, Justin, Akash, Bryan

Acceleration and Velocity with and without resistance:

- Classical:
  Section 2.3 Problem # 9: A motorboat weighs 32,000 lb and its motor provides a thrust of 5000 lb. Assume that the water resistance is 100 pounds for each foot per second of the speed v of the boat. Then $1000dv/dt = 5000 - 100v$: If the boat starts from rest, what is the maximum velocity that it can attain?(Akash Desai)

  Section 2.3 Problem # 10: A woman bails out of an airplane at an altitude of 10,000 ft, falls freely for 20 s, then opens her parachute. How long will it take her to reach the ground? Assume linear air resistance $\rho v\ ft/s2$, taking ρ = 0.15 without the parachute and ρ = 1.5 with the parachute.(Akash Desai)

- Laplace:

Section 2.3 Problem # 7 :
Suppose that a car starts from rest, its engine providing an acceleration of 10 $ft/s2$, while air resistance provides 0.1 $ft/s2$ of deceleration for each foot per second of the car's velocity. (a) Find the car's maximum possible (limiting) velocity. (b) Find how long it takes the car to attain 90% of its limiting velocity, and how far it travels while doing so.(Bryan Griffin)(Classical)

Section 1.2 Problem #26:

A projectile is fired straight upward with an initial velocity of 100 m=s from the top of a building 20 m high and falls to the ground at the base of the building. Find (a) its maximum height above the ground; (b) when it passes the top of the building; (c) its total time in the air. (*Bryan*)(Classical)

Section 1.2 Problem #33:

On the planet Gzyx, a ball dropped from a height of 20 ft hits the ground in 2 s. If a ball is dropped from the top of a 200-ft-tall building on Gzyx, how long will it take to hit the ground? With what speed will it hit? (*Bryan Griffin*)(Classical)

Acceleration and Velocity Applications with First and Second Order Equations

- Classical Methods:

Section 2.3 Problem # 17: Consider the crossbow bolt, shot straight upward from the ground ($\boldsymbol{y = 0}$) at time $\boldsymbol{t = 0}$ with initial velocity $v(0) = 49\ m/s$. Take $g = 9.8\frac{m}{s2}$ and ρ=0011 in Eq. (12). Then use Eqs. (13) and (14) to show that the bolt reaches its maximum height of about 108.47 m in about 4.61s.(*Justin*)(Classical)

Section 2.3 Problem #22

22. Suppose that $\rho(0) = 0.075$ (in fps units, with $g = 32\ ft/s2$) in Eq. (15) for a paratrooper falling with parachute open. If he jumps from an altitude of 10,000 ft and opens his parachute immediately, what will be his terminal speed? How long will it take him to reach the ground?(*Justin Peek*)(Classical)